AP CALCULUS AB

FRQ Tips and Tricks

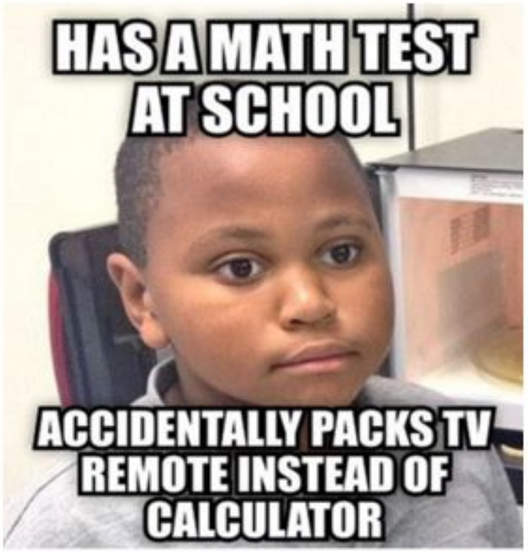
Exam Format

Section I- Multiple Choice (4 answer choices)		
Part A	30 MCQs without a calculator for 60 minutes	2 min/question
Part B	15 MCQs with a calculator for 45 minutes	3 min/question
Section II- Free Response		
Part A	2 FRQs with a calculator for 30 minutes	15 min/question
Part B	4 FRQs without a calculator for 60 minutes	15 min/question

Total time: 3 hours and 15 min.

Notes:

- Your work on the Multiple Choice is NOT scored.
- Showing your work is necessary in the Free Response.



Scoring

- Exam scores out of 108 points
- Each Multiple Choice is worth 1.2 points
  - 54 points
- Each FRQ is worth 9 points
  - 54 points

AP Score Conversion Chart
Calculus AB

Composite Score Range	AP Score
67-108	5
55-66	4
42-54	3
35-41	2
0-34	1

*Sample (each year is slightly different)

What does it take to pass?

- For a 5: ~ 67/108 = 62%
- For a 4: ~ 55/108 = 51%
- For a 3: ~ 42/108 = 39%

AP Score Conversion Chart
Calculus AB

Composite Score Range	AP Score
67-108	5
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35-41	2
0-34	1

*Sample (each year is slightly different)

GOAL IS JUST TO PASS :D

A Note on Calculator Use

- Not all question on the calculator section require a calculator
- MUST BE in **RADIANS**

When to use Calculator:

1. Plot the graph of a function
2. FInd the zeros of a function
3. Numerically calculate derivative of a function (at a point)
4. Numerically calculate the value of a definite integral

Showing Work on a Calculator FRQ

Question: Find the zeros of f(x)

$x^2+5x+6=0$ } calculator setup

$x=-3$
$x=-2$ } answer

Showing Work in General should be Logical

Show the reader what you plugged in:

Good: $f(2) = (2)^2 + 4$ Bad: $f(x) = (2)^2 + 4$

TIP #1: Save time where you can

- Do not use visual cues or Algebra 2 wording
- Do not box answer or cross out answers
    - Readers are told to not score this
- If you don't know part a, continue to part b
- Readers will follow wrong answers to give you points in remaining sections

Abbreviations are okay!

- b/c
- IVT
- MVT
- Misspelling are okay!

TIP #2: Work must be in the right area

Don't put work where it is not needed. If you found a derivative in part a, then realize that it was not needed in part a, but it is needed in part b, you will not receive any credit for the derivative. You must either, copy your work from part a into part b, or draw an arrow from the needed work in part a to part b. The readers want to make sure you know what work is needed for each part.

Answer QUESTION 1 PARTS A and B on this page.

PART A

PART B

Answer QUESTION 1 PARTS C and D on this page.

PART C

PART D

Use a pencil or a pen with black or dark blue ink. Do NOT write your name. Do NOT write outside the box.

TIP #3: Leave answers unsimplified

- If you can plug into a scientific calculator, you can leave it plugged in
  - Why? Saves time and avoid arithmetic mistakes
- This is really good for riemann sums
- Examples: sin(4), 2^2, 1+3

Example

(C) Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate $\int_0^9 r(t)\,dt$. Using correct units, explain the meaning of $\int_0^9 r(t)\,dt$ in the context of the problem.

$$\int_0^9 r(t)\,dt \approx (3-0)\cdot r(0) + (5-3)\cdot r(3) + (6-5)\cdot r(5) + (9-6)\cdot r(6)$$

$$= 3(72) + 2(95) + 1(112) + 3(77) = 749$$

Stop here

$\int_0^9 r(t)\,dt$ is the total number of rotations of the wheel of the stationary bicycle over the time interval $0 \le t \le 9$ minutes.

TIP #4: Use PROPER function NAMES

- Refer to the specific function/derivative by name such as f(x) or k'(x)
- **NO-NO Words**: "the function" or "it", "the derivative", or "the slope"
- Tip: answer the questions in a full sentence.

Example

When is g(x) increasing? Justify your answer.

- g(x) is increasing (1,2) b/c g'(x)>0

TIP #5: Justify with Calculus Reasons

- Do not use visual cues or Algebra 2 wording
- Keep it SHORT

Example

Find the intervals of increase for f(x). Justify your answer.

Good answer: "f(x) is increasing on (1,3) b/c $f'(x)>0$"

Bad answer: "f(x) is increasing on (1,3) b/c $f'(x)$ is above x-axis"

TIP #6: Theorems and L'hopital's Work

- Mention Theorems like l'hopital's, IVT, MVT
- For l'hopital's you MUST separate numerator and denominator
- Do not drop off LIM notation until you plug into

Example

$$\lim_{x\to 2}\frac{x-2}{x^2-4} =$$

$$\lim_{x\to 2} x-2 = 0$$

$$\lim_{x\to 2} x^2-4 = 0$$

indet.

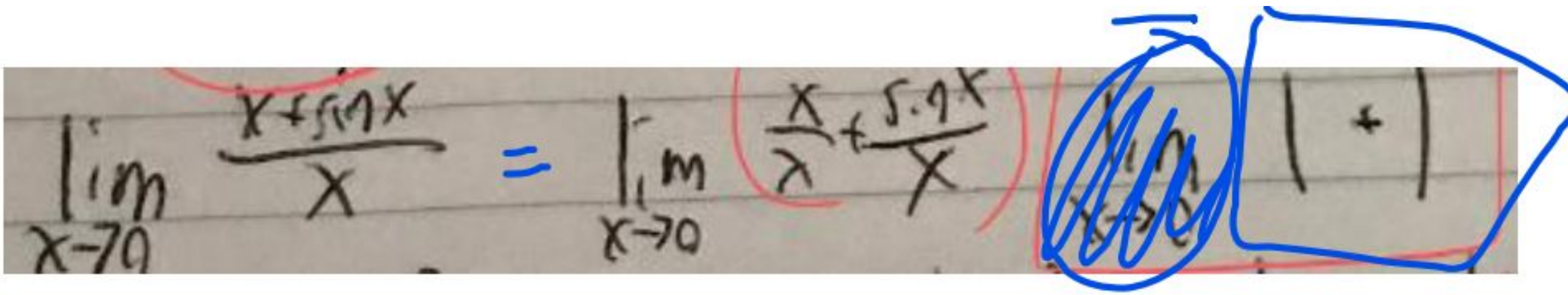
By l'hopital's,

$$= \lim_{x\to 2}\frac{1}{2x} = \frac{1}{4}$$

TIP #7: Avoid Linkage Issue

- Do not drop off LIM notation until you plug into

Example



TIP #8: Truncate to THREE decimal places

- Round/truncate at the LAST STEP
  - truncate>round
- THREE DECIMAL PLACES
  - They will take off point if you put 1.11 and the answer is 1.112

TIP #9: You Get Points for Set -up

Example

Find the absolute maximum of g. Justify your answer.

$g'(x)=0$ ← just thus gives you a point

TIP #10: A note on Absolute Min/Max Type questions

- Do not write answer as a coordinate point
- Answer the question,
  - If they ask for the max/min, give the value
  - If they ask for WHEN min/max occurs, give time x= or t=

Example

(C) $g'(x) = f(x) = 0 \Rightarrow x = 1, x = 3$

x	$g(x)$
-1	$\frac{7}{2}$
1	$\frac{11}{2}$
3	$\frac{9}{2}$
4	$\frac{13}{2}$

The absolute minimum value of g is $\frac{7}{2}$, and the absolute maximum value of g is $\frac{13}{2}$.

(C) On the closed interval $-1 \le x \le 4$, find the absolute minimum value of g and find the absolute maximum value of g. Justify your answers.

TIP #11: Approximation Questions

- If they ask for an approximation, use the approximation symbol
  - Otherwise you are saying it's equal and that would be incorrect
- Common: Riemann Sums, Trapezoidal, Linear Approximation
  - Make sure to show set-up: “shape width*height”

(C) Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate $\int_0^9 r(t)\,dt$. Using correct units, explain the meaning of $\int_0^9 r(t)\,dt$ in the context of the problem.

$$\int_0^9 r(t)\,dt \approx (3-0)\cdot r(0) + (5-3)\cdot r(3) + (6-5)\cdot r(5) + (9-6)\cdot r(6)$$

$$= 3(72) + 2(95) + 1(112) + 3(77) = 749$$

$\int_0^9 r(t)\,dt$ is the total number of rotations of the wheel of the stationary bicycle over the time interval $0 \le t \le 9$ minutes.

TIP #12: Connect Motion Derivatives

- If you want to use v(t) as velocity function connect derivative of position equal to v(t)

Example

$$v(t) = s'(t) = 6t^2 - 4t$$

TIP #13: Remember the units (if asked)

- Only include units if asked to.
- If asked, you a single point for just including the units
- If you're not sure, use the function description and consider if you're calculating an amount or a rate

t (minutes)	0	3	7	12
$C(t)$ (degrees Celsius)	100	85	69	55

1. The temperature of coffee in a cup at time t minutes is modeled by a decreasing differentiable function C, where $C(t)$ is measured in degrees Celsius. For $0 \le t \le 12$, selected values of $C(t)$ are given in the table shown.

   (a) Approximate $C'(5)$ using the average rate of change of C over the interval $3 \le t \le 7$. Show the work that leads to your answer and include units of measure.

NATIONAL MATH + SCIENCE INITIATIVE

Common Mistakes on the Free Response Portion

- Algebra and arithmetic mistakes
- Missing limits of integration
- Not considering endpoints of an interval for absolute extrema
- Not giving both coordinates of a point when required
- Giving both coordinates of a point when only one is asked for---value of a function refers to the y-coordinate
- Doing all the work of a problem but not answering the question. If the question is a "yes" or "no" question, show all needed work and then answer the question with a "yes" or a "no"
- Ignoring units of measure
- Making improper assumptions of a function when only a table of values is given
- Doing all work on a calculator without showing the reader what you are inputting into your calculator
- Rounding errors---you're answers need to be accurate to 3 decimal places. This means that you need to carry more than 3 decimal places in your work prior to your final answer. If you use values that are already rounded to 3 decimal places in your computations, you risk your final answer not having the required accuracy. Know how to store intermediate values on your calculator.

Multiple Choice Random Advice

Tips for the AP Exam by Mark Howell*

The following are some tips for the **multiple-choice** portion of the exam:

1. If you took the time to read a question and all the answer choices but decided to skip it, take an extra ten seconds and guess. There is no penalty for a wrong answer, so you should answer every question..
2. Don't waste time guessing randomly. Use the time to try to answer a question using your knowledge of Calculus,
3. If a common paragraph, graph, or table refers to a group of questions and you took the time to read it, try each question in the group.
4. Don't go back and change an answer unless you have found an error in your work. Your first impulse is more likely to be correct.
5. Some multiple choice questions can be answered by working backwards. Plug the answer choices into the problem and see which answer works out.
6. Don't get stuck. Skip a hard question and come back to it if you have time after finishing the rest. But be careful marking your answer sheet when you skip questions.
7. On the calculator portion, you won't need your calculator on all the questions. If all or some of the answers have three digits after the decimal point (e.g., 98.765), it's time to pick up the calculator (but, of course, there may be other times to use it as well).

Differential Equations FRQ

- Keep each step a different line

Example

(c) $\frac{1}{y^2}\,dy = \frac{1}{x-1}\,dx$

$\int \frac{1}{y^2}\,dy = \int \frac{1}{x-1}\,dx$

$-\frac{1}{y} = \ln|x-1| + C$

$-\frac{1}{3} = \ln|2-1| + C \Rightarrow C = -\frac{1}{3}$

$-\frac{1}{y} = \ln|x-1| - \frac{1}{3}$

$y = \dfrac{1}{\frac{1}{3} - \ln(x-1)}$

Note: This solution is valid for $1 < x < 1 + e^{1/3}$.

$5: \begin{cases} 1: \text{separation of variables} \\ 2: \text{antiderivatives} \\ 1: \text{constant of integration and} \\ \quad\ \text{uses initial condition} \\ 1: \text{solves for } y \end{cases}$

Note: max $3/5$ [1-2-0-0] if no constant of integration

Note: $0/5$ if no separation of variables

Q & A

What questions do you have about the AP exam?

SCORING PRACTICE

You've been hired as a AP Reader!

Question 1

With Calculator

t (minutes)	0	3	7	12
$C(t)$ (degrees Celsius)	100	85	69	55

The temperature of coffee in a cup at time t minutes is modeled by a decreasing differentiable function C, where $C(t)$ is measured in degrees Celsius. For $0 \le t \le 12$, selected values of $C(t)$ are given in the table shown.

2024 Question 1a

t (minutes)	0	3	7	12
$C(t)$ (degrees Celsius)	100	85	69	55

The temperature of coffee in a cup at time t minutes is modeled by a decreasing differentiable function C, where $C(t)$ is measured in degrees Celsius. For $0 \le t \le 12$, selected values of $C(t)$ are given in the table shown.

(a) Approximate $C'(5)$ using the average rate of change of C over the interval $3 \le t \le 7$. Show the work that leads to your answer and include units of measure.

Q1a 2024 - Answer

Model Solution	Scoring	
(a) Approximate $C'(5)$ using the average rate of change of C over the interval $3 \le t \le 7$. Show the work that leads to your answer and include units of measure.		
$C'(5) \approx \dfrac{C(7) - C(3)}{7 - 3} = \dfrac{69 - 85}{4} = -4$ degrees Celsius per minute	Estimate with supporting work	**1 point**
	Units	**1 point**

Scoring notes:

- To earn the first point a response must include a difference and a quotient as the supporting work.
- $\dfrac{-16}{7-3}$, $\dfrac{69-85}{7-3}$, or $\dfrac{69-85}{4}$ is sufficient to earn the first point.
- A response that presents only units without a numerical approximation for $C'(5)$ does not earn the second point.
- The second point is also earned for "degrees per minute" attached to a numerical value.

Total for part (a) **2 points**

Q1a 2024 - Sample 1A

Answer

$C'(5) \approx \frac{C(7)-C(3)}{7-3} = \frac{69-85}{4} = -4$ degrees Celsius per minute	Estimate with supporting work	1 point
	Units	1 point

Sample: How many points would you award?

Answer QUESTION 1 parts (a) and (b) on this page.

t (minutes)	0	3	7	12
$C(t)$ (degrees Celsius)	100	85	69	55

Response for question 1(a)

$C`(5) = \frac{69-85}{7-3} = -4$ degrees Celsius per minute

$\frac{f(b)-f(a)}{b-a}$

Actual Score: 2

Q1a 2024 - Sample 1B

Answer

$C'(5) \approx \frac{C(7)-C(3)}{7-3} = \frac{69-85}{4} = -4$ degrees Celsius per minute	Estimate with supporting work	1 point
	Units	1 point

Sample: How many points would you award?

1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1

Answer QUESTION 1 parts (a) and (b) on this page.

t (minutes)	0	3	7	12
$C(t)$ (degrees Celsius)	100	85	69	55

Response for question 1(a)

$$\frac{f(b)-f(a)}{b-a} = \frac{69-85}{7-3} = \frac{-16}{4} = -4$$

MVT $\frac{7+3}{2} = 10$ $\quad C'(5) \approx -4$ degrees C°/minute

Actual Score: 2

Q1a 2024 - Sample 1c

Answer

$C'(5) \approx \frac{C(7)-C(3)}{7-3} = \frac{69-85}{4} = -4$ degrees Celsius per minute	Estimate with supporting work	1 point
	Units	1 point

Sample: How many points would you award?

1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1

Answer QUESTION 1 parts (a) and (b) on this page.

t (minutes)	0	3	7	12
$C(t)$ (degrees Celsius)	100	85	69	55

Response for question 1(a)

$\frac{69-85}{7-3} = \frac{-16}{4} = -4$ degrees celcius

Actual Score: 1

2024 Question 1b

t (minutes)	0	3	7	12
$C(t)$ (degrees Celsius)	100	85	69	55

The temperature of coffee in a cup at time t minutes is modeled by a decreasing differentiable function C, where $C(t)$ is measured in degrees Celsius. For $0 \le t \le 12$, selected values of $C(t)$ are given in the table shown.

(b) Use a left Riemann sum with the three subintervals indicated by the data in the table to approximate the value of $\int_0^{12} C(t)\,dt$. Interpret the meaning of $\frac{1}{12}\int_0^{12} C(t)\,dt$ in the context of the problem.

2024 Question 1b - Answer

(b) Use a left Riemann sum with the three subintervals indicated by the data in the table to approximate the value of $\int_0^{12} C(t)\,dt$. Interpret the meaning of $\frac{1}{12}\int_0^{12} C(t)\,dt$ in the context of the problem.

$\int_0^{12} C(t)\,dt \approx (3-0)\cdot C(0) + (7-3)\cdot C(3) + (12-7)\cdot C(7)$ $= 3\cdot 100 + 4\cdot 85 + 5\cdot 69 = 985$	Form of left Riemann sum	**1 point**
	Estimate	**1 point**
$\frac{1}{12}\int_0^{12} C(t)\,dt$ is the average temperature of the coffee (in degrees Celsius) over the interval from $t = 0$ to $t = 12$.	Interpretation	**1 point**

Scoring notes:

- Read "$=$" as "$\approx$" for the first point.
- To earn the first point at least five of the six factors in the Riemann sum must be correct. If any of the six factors is incorrect, the response does not earn the second point.
- A response of $(3-0)\cdot C(0) + (7-3)\cdot C(3) + (12-7)\cdot C(7)$ earns the first point. Values must be pulled from the table to earn the second point.
- A response of $3\cdot 100 + 4\cdot 85 + 5\cdot 69$ earns both the first and second points, unless there is a subsequent error in simplification, in which case the response would earn only the first point.
- A completely correct right Riemann sum (e.g., $3\cdot 85 + 4\cdot 69 + 5\cdot 55$) earns 1 of the first 2 points. An unsupported answer of 806 does not earn either of the first 2 points.
- Units will not affect scoring for the second point.
- To earn the third point the interpretation must include both "average temperature" and the time interval. The response need not include a reference to units. However, if incorrect units are given in the interpretation, the response does not earn the third point.

2024 Sample 1b

Answer

$\int_0^{12} C(t)\,dt \approx (3-0)\cdot C(0) + (7-3)\cdot C(3) + (12-7)\cdot C(7)$ $= 3\cdot 100 + 4\cdot 85 + 5\cdot 69 = 985$	Form of left Riemann sum	**1 point**
	Estimate	**1 point**
$\frac{1}{12}\int_0^{12} C(t)\,dt$ is the average temperature of the coffee (in degrees Celsius) over the interval from $t = 0$ to $t = 12$.	Interpretation	**1 point**

Sample: How many points would you award?

Response for question 1(b)

3(100) + 4(85) + 5(69) = 985 degrees Celsius

$\frac{1}{12}\int_0^{12}$ C(t) is the average temperature over the interval from 0 min to 12 min

Actual Score: 3

2024 Sample 2b

Answer

$\int_0^{12} C(t)\,dt \approx (3-0)\cdot C(0) + (7-3)\cdot C(3) + (12-7)\cdot C(7)$ $= 3\cdot 100 + 4\cdot 85 + 5\cdot 69 = 985$	Form of left Riemann sum	**1 point**
	Estimate	**1 point**
$\frac{1}{12}\int_0^{12} C(t)\,dt$ is the average temperature of the coffee (in degrees Celsius) over the interval from $t = 0$ to $t = 12$.	Interpretation	**1 point**

Sample: How many points would you award?

Response for question 1(b)

$\int_0^{12} c(t)\,dt$

$\approx 3(100) + 4(85) + 5(69)$

≈ 985

$\frac{1}{12}\int_0^{12} c(t)\,dt$

$\approx 82.08333333\ °C$

This is the average temperature of the coffee from 0 to 12 minutes

Actual Score: 3

2024 Sample 3b

Answer

$\int_0^{12} C(t)\,dt \approx (3-0)\cdot C(0) + (7-3)\cdot C(3) + (12-7)\cdot C(7)$ $= 3\cdot 100 + 4\cdot 85 + 5\cdot 69 = 985$	Form of left Riemann sum	1 point
	Estimate	1 point
$\frac{1}{12}\int_0^{12} C(t)\,dt$ is the average temperature of the coffee (in degrees Celsius) over the interval from $t = 0$ to $t = 12$.	Interpretation	1 point

Sample: How many points would you award?

Response for question 1(b)

$(3\cdot 100) + (4\cdot 85) + (5\cdot 69)$

$300 + 340 + 345$

985

$\frac{1}{12}\int_0^{12} C(t)$ is the average rate of changes for degrees celcius over 12 minutes of time

Actual Score: 2

2024 Question 1c

t (minutes)	0	3	7	12
$C(t)$ (degrees Celsius)	100	85	69	55

The temperature of coffee in a cup at time t minutes is modeled by a decreasing differentiable function C, where $C(t)$ is measured in degrees Celsius. For $0 \le t \le 12$, selected values of $C(t)$ are given in the table shown.

(c) For $12 \le t \le 20$, the rate of change of the temperature of the coffee is modeled by $C'(t) = \frac{-24.55e^{0.01t}}{t}$, where $C'(t)$ is measured in degrees Celsius per minute. Find the temperature of the coffee at time $t = 20$. Show the setup for your calculations.

2024 Question 1c - Answer

(c) For $12 \le t \le 20$, the rate of change of the temperature of the coffee is modeled by $C'(t) = \frac{-24.55e^{0.01t}}{t}$, where $C'(t)$ is measured in degrees Celsius per minute. Find the temperature of the coffee at time $t = 20$. Show the setup for your calculations.

$C(20) = C(12) + \int_{12}^{20} C'(t)\,dt$	Integral	**1 point**
	Uses initial condition	**1 point**
$= 55 - 14.670812 = 40.329188$ The temperature of the coffee at time $t = 20$ is 40.329 degrees Celsius.	Answer	**1 point**

2024 Question 1c - Answer Con't

Scoring notes:

- The first point is earned for a definite integral with integrand $C'(t)$. If the limits of integration are incorrect, the response does not earn the third point.
- A linkage error such as $C(20) = \int_{12}^{20} C'(t)\,dt = 55 - 14.670812$ or $\int_{12}^{20} C'(t)\,dt = -14.670812 = 40.329188$ earns the first 2 points but does not earn the third point.
- Missing differential (dt):
    - Unambiguous responses of $C(20) = C(12) + \int_{12}^{20} C'(t)$ or $C(20) = 55 + \int_{12}^{20} C'(t)$ earn the first 2 points and are eligible for the third point.
    - Ambiguous responses of $C(20) = \int_{12}^{20} C'(t) + C(12)$ or $C(20) = \int_{12}^{20} C'(t) + 55$ do not earn the first point, earn the second point, and earn the third point if the given numeric answer is correct. If there is no numeric answer given, these responses do not earn the third point.
- The second point is earned for adding $C(12)$ or 55 to a definite integral with a lower limit of 12, either symbolically or numerically.
- The third point is earned for an answer of $55 - 14.671$ or $-14.671 + 55$ with no additional simplification, provided there is some supporting work for these values.
- An answer of just 40.329 with no supporting work does not earn any points.

2024 Sample 1c

Answer

$C(20) = C(12) + \int_{12}^{20} C'(t)\,dt$	Integral	**1 point**
	Uses initial condition	**1 point**
$= 55 - 14.670812 = 40.329188$ The temperature of the coffee at time $t = 20$ is 40.329 degrees Celsius.	Answer	**1 point**

Sample: How many points would you award?

Response for question 1(c)

$$55 + \int_{12}^{20} C`(t)\,dt = 40.329 \text{ degrees Celsius}$$

Actual Score: 3

2024 Sample 2c

Answer

$C(20) = C(12) + \int_{12}^{20} C'(t)\,dt$	Integral	**1 point**
	Uses initial condition	**1 point**
$= 55 - 14.670812 = 40.329188$ The temperature of the coffee at time $t = 20$ is 40.329 degrees Celsius.	Answer	**1 point**

Sample: How many points would you award?

Response for question 1(c)

$$c'(t) = \frac{-24.55e^{0.01t}}{t}$$

$$\int_{12}^{20} \frac{-24.55e^{0.01t}}{t}$$

$$= -14.67081194$$

Actual Score: 1

2024 Sample 3c

Answer

$C(20) = C(12) + \int_{12}^{20} C'(t)\,dt$	Integral	**1 point**
	Uses initial condition	**1 point**
$= 55 - 14.670812 = 40.329188$ The temperature of the coffee at time $t = 20$ is 40.329 degrees Celsius.	Answer	**1 point**

Sample: How many points would you award?

Response for question 1(c)

$$C'(20) = \frac{-24.55e^{0.01(20)}}{20}$$

$$\approx -1.499 \text{ celcus}$$

Actual Score: 0

2024 Question 1d

t (minutes)	0	3	7	12
$C(t)$ (degrees Celsius)	100	85	69	55

The temperature of coffee in a cup at time t minutes is modeled by a decreasing differentiable function C, where $C(t)$ is measured in degrees Celsius. For $0 \le t \le 12$, selected values of $C(t)$ are given in the table shown.

(d) For the model defined in part (c), it can be shown that $C''(t) = \dfrac{0.2455e^{0.01t}(100 - t)}{t^2}$. For $12 < t < 20$, determine whether the temperature of the coffee is changing at a decreasing rate or at an increasing rate. Give a reason for your answer.

2024 Question 1d - Answer

(d) For the model defined in part (c), it can be shown that $C''(t) = \dfrac{0.2455e^{0.01t}(100-t)}{t^2}$. For $12 < t < 20$, determine whether the temperature of the coffee is changing at a decreasing rate or at an increasing rate. Give a reason for your answer.

Because $C''(t) > 0$ on the interval $12 < t < 20$, the rate of change in the temperature of the coffee, $C'(t)$, is increasing on this interval. That is, on the interval $12 < t < 20$, the temperature of the coffee is changing at an increasing rate.	Answer with reason	**1 point**

Scoring notes:

- This point is earned only for a correct answer with a correct reason that references the sign of the second derivative of C.
- A response that provides a reason based on the evaluation of $C''(t)$ at a single point does not earn this point.
- A response that uses ambiguous pronouns (such as "It is positive, so increasing") does not earn this point.
- A response does not need to reference the interval $12 < t < 20$ to earn the point.

Total for part (d) **1 point**

2024 Sample 1d

Answer

Because $C''(t) > 0$ on the interval $12 < t < 20$, the rate of change in the temperature of the coffee, $C'(t)$, is increasing on this interval. That is, on the interval $12 < t < 20$, the temperature of the coffee is changing at an increasing rate.	Answer with reason	**1 point**

Sample: How many points would you award?

Response for question 1(d)

The temp of the coffee is changing at an increasing rate because C``(t) is + over the interval 12<t<20

Actual Score: 1

2024 Sample 2d

Answer

Because $C''(t) > 0$ on the interval $12 < t < 20$, the rate of change in the temperature of the coffee, $C'(t)$, is increasing on this interval. That is, on the interval $12 < t < 20$, the temperature of the coffee is changing at an increasing rate.	Answer with reason	**1 point**

Sample: How many points would you award?

Response for question 1(d)

$$c''(t) = \frac{0.2455e^{0.01t}(100-t)}{t^2}$$

$$c''(16) = \frac{0.2455e^{0.01(16)}(100-16)}{(16)^2}$$

$$= 0.094531801 5$$

Increasing rate since $c''(16)$ is positive and in the given interval.

Actual Score: 0

2024 Sample 3d

Answer

Because $C''(t) > 0$ on the interval $12 < t < 20$, the rate of change in the temperature of the coffee, $C'(t)$, is increasing on this interval. That is, on the interval $12 < t < 20$, the temperature of the coffee is changing at an increasing rate.	Answer with reason	**1 point**

Sample: How many points would you award?

Response for question 1(d)

$C''(t)$ is changing at an increasing rate as the equation $\int_{12}^{20} C''(t)\,dt$ ends up as a positive rate

Actual Score: 0

Question 3

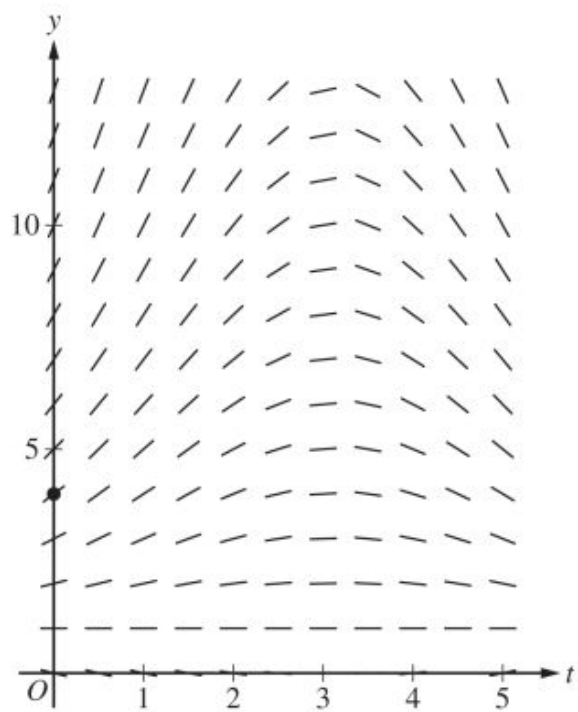
With NO Calculator

The depth of seawater at a location can be modeled by the function H that satisfies the differential equation $\frac{dH}{dt} = \frac{1}{2}(H - 1)\cos\left(\frac{t}{2}\right)$, where $H(t)$ is measured in feet and t is measured in hours after noon ($t = 0$). It is known that $H(0) = 4$.

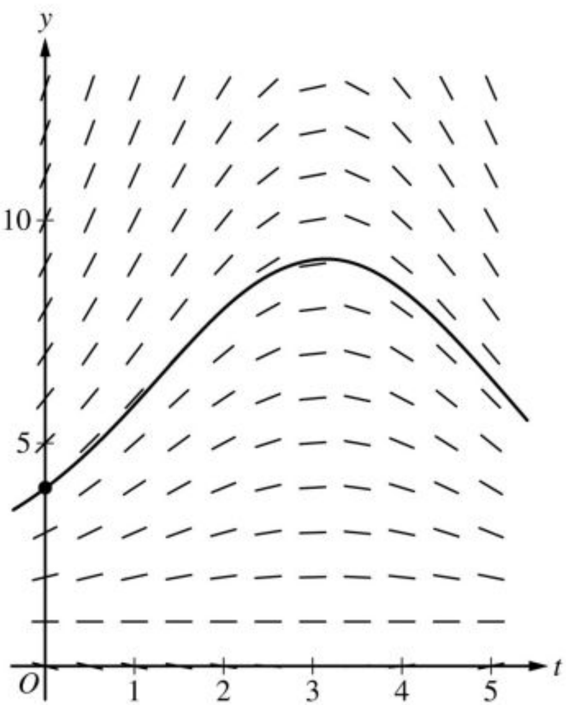
2024 Question 3a

The depth of seawater at a location can be modeled by the function H that satisfies the differential equation $\frac{dH}{dt} = \frac{1}{2}(H-1)\cos\left(\frac{t}{2}\right)$, where $H(t)$ is measured in feet and t is measured in hours after noon ($t = 0$). It is known that $H(0) = 4$.

(a) A portion of the slope field for the differential equation is provided. Sketch the solution curve, $y = H(t)$, through the point $(0, 4)$.



Q3a 2024 - Answer

Model Solution	Scoring
(a) A portion of the slope field for the differential equation is provided. Sketch the solution curve, $y = H(t)$, through the point $(0, 4)$.	
	Solution curve **1 point**

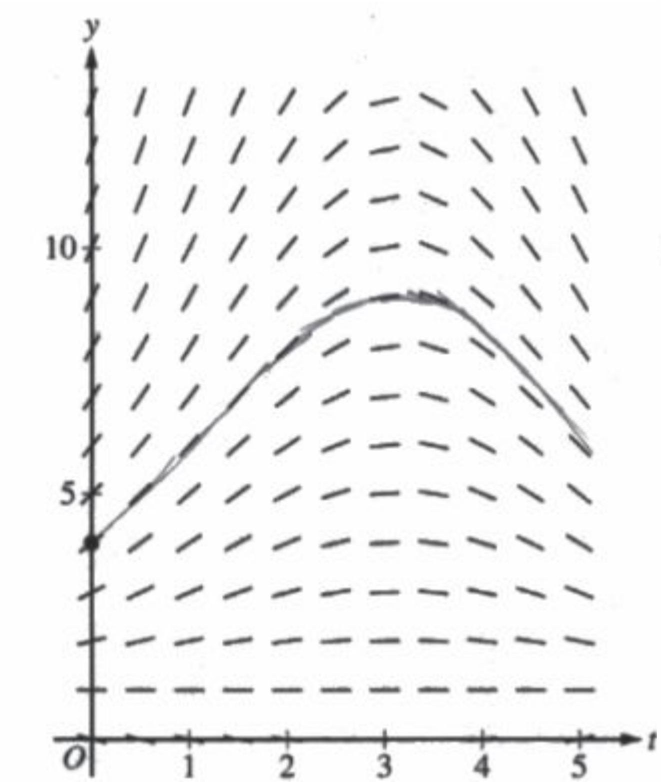
Scoring notes:

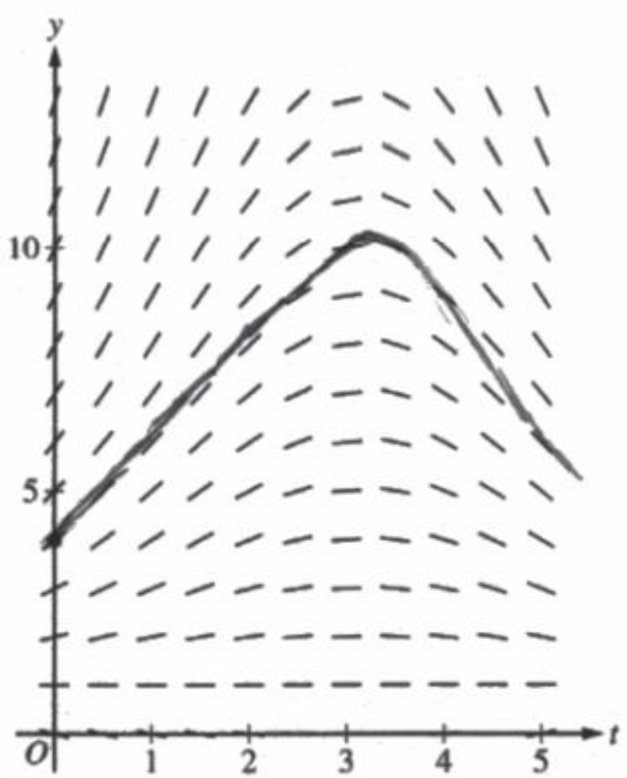
- The solution curve must pass through the point $(0, 4)$, extend to at least $t = 4.5$, and have no obvious conflicts with the given slope lines.
- Only portions of the solution curve within the given slope field are considered.

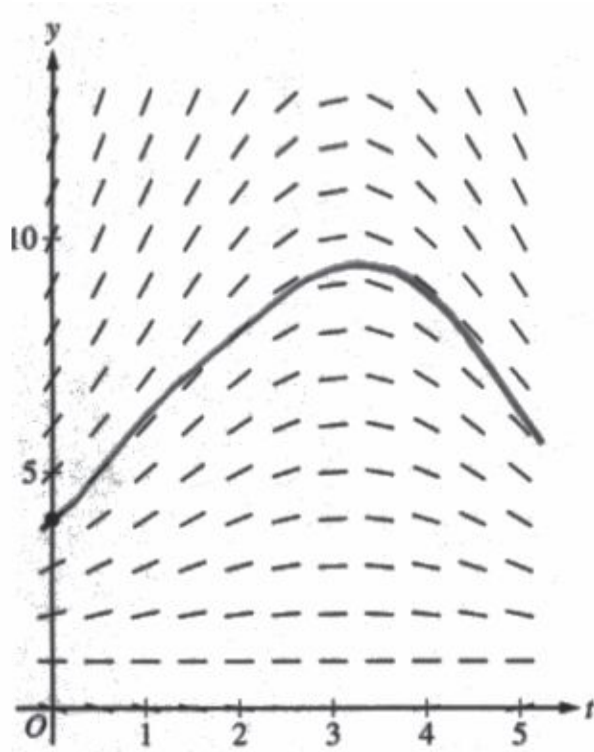
Total for part (a) 1 point

Q3a 2024 - Sample 1a, 2a, and 3a

Samples: How many points would you award?







Actual Score: 1

Actual Score: 1

Actual Score: 1

2024 Question 3b

The depth of seawater at a location can be modeled by the function H that satisfies the differential equation $\frac{dH}{dt} = \frac{1}{2}(H-1)\cos\left(\frac{t}{2}\right)$, where $H(t)$ is measured in feet and t is measured in hours after noon ($t = 0$). It is known that $H(0) = 4$.

(b) For $0 < t < 5$, it can be shown that $H(t) > 1$. Find the value of t, for $0 < t < 5$, at which H has a critical point. Determine whether the critical point corresponds to a relative minimum, a relative maximum, or neither a relative minimum nor a relative maximum of the depth of seawater at the location. Justify your answer.

Q3b 2024 - Answer

(b) For $0 < t < 5$, it can be shown that $H(t) > 1$. Find the value of t, for $0 < t < 5$, at which H has a critical point. Determine whether the critical point corresponds to a relative minimum, a relative maximum, or neither a relative minimum nor a relative maximum of the depth of seawater at the location. Justify your answer.

Because $H(t) > 1$, then $\frac{dH}{dt} = 0$ implies $\cos\left(\frac{t}{2}\right) = 0$.	Considers sign of $\frac{dH}{dt}$	**1 point**
This implies that $t = \pi$ is a critical point.	Identifies $t = \pi$	**1 point**
For $0 < t < \pi$, $\frac{dH}{dt} > 0$ and for $\pi < t < 5$, $\frac{dH}{dt} < 0$. Therefore, $t = \pi$ is the location of a relative maximum value of H.	Answer with justification	**1 point**

Q3b 2024 - Answer

Scoring notes:

- The first point is earned for considering $\frac{dH}{dt} = 0$, $\frac{dH}{dt} > 0$, $\frac{dH}{dt} < 0$, $\cos\left(\frac{t}{2}\right) = 0$, $\cos\left(\frac{t}{2}\right) > 0$, or $\cos\left(\frac{t}{2}\right) < 0$.
- The second point is earned for identifying $t = \pi$, with or without supporting work. A response may consider $H = 1$ or $t = 1$ as potential critical points without penalty.
- The third point cannot be earned without the first point. The third point is earned only for a correct justification and a correct answer of "relative maximum."
- The justification can be shown by determining the sign of $\frac{dH}{dt}$ (or $\cos\left(\frac{t}{2}\right)$) at a single value in $0 < t < \pi$ and at a single value in $\pi < t < 5$. It is not necessary to state that $\frac{dH}{dt}$ does not change sign on these intervals.
- The third point can also be earned by using the Second Derivative Test. For example:

$$\frac{d^2H}{dt^2} = \frac{1}{2}(H-1)\left(-\frac{1}{2}\sin\left(\frac{t}{2}\right)\right) + \cos\left(\frac{t}{2}\right)\cdot\frac{1}{2}\cdot\frac{dH}{dt}$$

$$\left.\frac{d^2H}{dt^2}\right|_{t=\pi} < 0$$

Therefore, $t = \pi$ is the location of a relative maximum value of H.

Total for part (b) 3 points

2024 Sample 1b

Answer

Because $H(t) > 1$, then $\frac{dH}{dt} = 0$ implies $\cos\left(\frac{t}{2}\right) = 0$.	Considers sign of $\frac{dH}{dt}$	**1 point**
This implies that $t = \pi$ is a critical point.	Identifies $t = \pi$	**1 point**
For $0 < t < \pi$, $\frac{dH}{dt} > 0$ and for $\pi < t < 5$, $\frac{dH}{dt} < 0$. Therefore, $t = \pi$ is the location of a relative maximum value of H.	Answer with justification	**1 point**

Sample: How many points would you award?

Response for question 3(b)

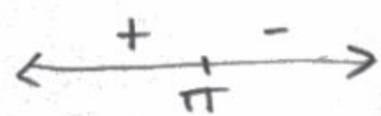
$H > 1$

$\frac{dH}{dt} = 0$

$0 = \frac{1}{2}(H-1)\cos\left(\frac{t}{2}\right)$

$0 = \cos\left(\frac{t}{2}\right)$

$t = \pi, 3\pi$



H has a critical point at π on the interval $0 < t < 5$, and the point is a relative maximum because H' changes from positive to negative at $t = \pi$.

Actual Score: 3

2024 Sample 2b

Answer

Because $H(t) > 1$, then $\frac{dH}{dt} = 0$ implies $\cos\left(\frac{t}{2}\right) = 0$. This implies that $t = \pi$ is a critical point.	Considers sign of $\frac{dH}{dt}$	1 point
	Identifies $t = \pi$	1 point
For $0 < t < \pi$, $\frac{dH}{dt} > 0$ and for $\pi < t < 5$, $\frac{dH}{dt} < 0$. Therefore, $t = \pi$ is the location of a relative maximum value of H.	Answer with justification	1 point

Sample: How many points would you award?

Response for question 3(b)

$$\frac{dH}{dt} = \frac{1}{2}(H-1)\cos\left(\frac{t}{2}\right)$$

$$\frac{dH}{dt} = \frac{1}{2}(H-1)\cos\left(\frac{t}{2}\right)$$

$$\frac{dH}{dt} = \frac{1}{2}(4-1)\cos(0)$$

$$\frac{dH}{dt} = \frac{3}{2}$$

$$0 = \frac{1}{2}(H-1)\cos\left(\frac{t}{2}\right)$$

H has no critical point.

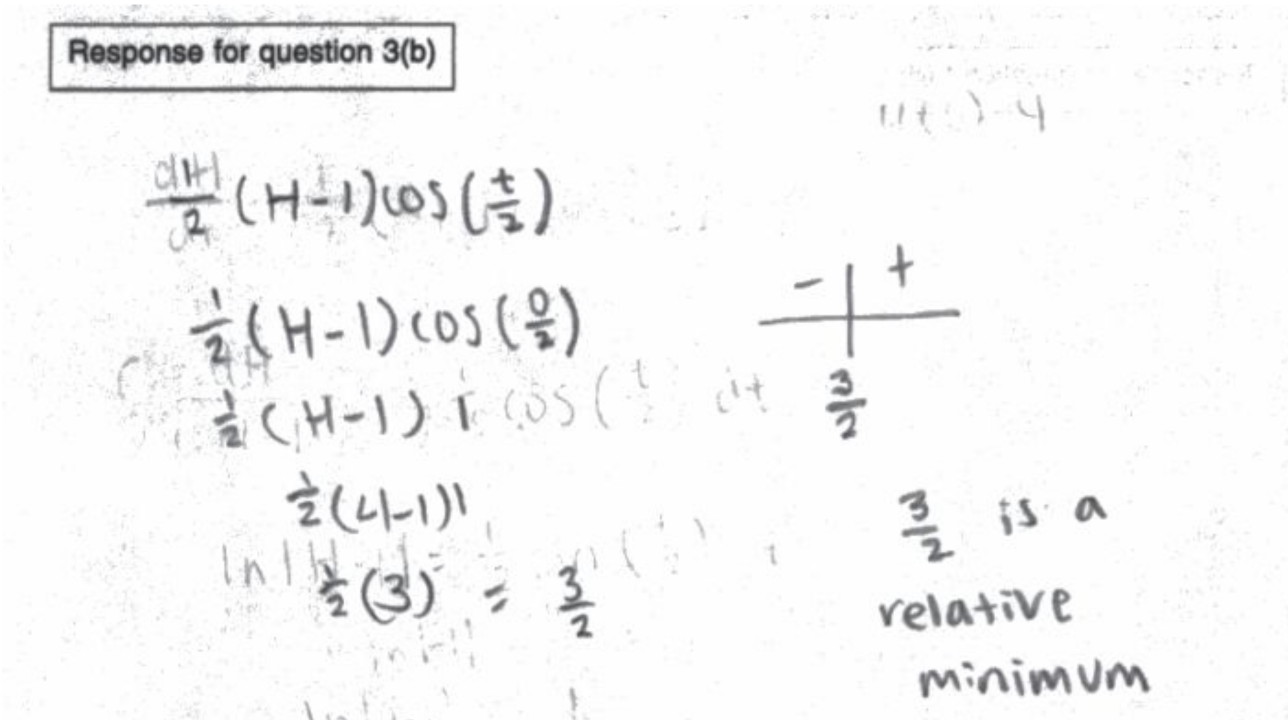
Actual Score: 1

2024 Sample 3b

Answer

Because $H(t) > 1$, then $\frac{dH}{dt} = 0$ implies $\cos\left(\frac{t}{2}\right) = 0$.	Considers sign of $\frac{dH}{dt}$	**1 point**
This implies that $t = \pi$ is a critical point.	Identifies $t = \pi$	**1 point**
For $0 < t < \pi$, $\frac{dH}{dt} > 0$ and for $\pi < t < 5$, $\frac{dH}{dt} < 0$. Therefore, $t = \pi$ is the location of a relative maximum value of H.	Answer with justification	**1 point**

Sample: How many points would you award?


Response for question 3(b)

$\frac{dH}{d2}(H-1)\cos\left(\frac{t}{2}\right)$

$\frac{1}{2}(H-1)\cos\left(\frac{0}{2}\right)$

$\frac{1}{2}(H-1)\,1$

$\frac{1}{2}(4-1)1$

$\frac{1}{2}(3) = \frac{3}{2}$

$\frac{3}{2}$ is a relative minimum

Actual Score: 0

2024 Question 3c

The depth of seawater at a location can be modeled by the function H that satisfies the differential equation $\frac{dH}{dt} = \frac{1}{2}(H - 1)\cos\left(\frac{t}{2}\right)$, where $H(t)$ is measured in feet and t is measured in hours after noon ($t = 0$). It is known that $H(0) = 4$.

(c) Use separation of variables to find $y = H(t)$, the particular solution to the differential equation $\frac{dH}{dt} = \frac{1}{2}(H - 1)\cos\left(\frac{t}{2}\right)$ with initial condition $H(0) = 4$.

Q3c 2024 - Answer

(c) Use separation of variables to find $y = H(t)$, the particular solution to the differential equation $\frac{dH}{dt} = \frac{1}{2}(H-1)\cos\left(\frac{t}{2}\right)$ with initial condition $H(0) = 4$.

$\frac{dH}{H-1} = \frac{1}{2}\cos\left(\frac{t}{2}\right)dt$	Separation of variables	**1 point**
$\int \frac{dH}{H-1} = \int \frac{1}{2}\cos\left(\frac{t}{2}\right)dt$ $\Rightarrow \ln\lvert H-1\rvert = \sin\left(\frac{t}{2}\right) + C$	One antiderivative	**1 point**
	Second antiderivative	**1 point**
$\ln\lvert 4-1\rvert = \sin\left(\frac{0}{2}\right) + C \Rightarrow C = \ln 3$ Because $H(0) = 4$, $H > 1$, so $\lvert H-1\rvert = H-1$. $\ln(H-1) = \sin\left(\frac{t}{2}\right) + \ln 3$	Constant of integration and uses initial condition	**1 point**
$H - 1 = e^{\sin(t/2)+\ln 3} = 3e^{\sin(t/2)}$ $H(t) = 1 + 3e^{\sin(t/2)}$	Solves for H	**1 point**

Q3c 2024 - Answer

Scoring notes:

- A response with no separation of variables earns 0 out of 5 points.
- A response that presents $\int \frac{dH}{H-1} = \ln(H-1)$ without absolute value symbols earns that antiderivative point.
- A response with no constant of integration can earn at most the first 3 points.
- A response is eligible for the fourth point only if it has earned the first point and at least 1 of the 2 antiderivative points.
- An eligible response earns the fourth point by correctly including the constant of integration in an equation and substituting 0 for t and 4 for H.
- A response is eligible for the fifth point only if it has earned the first 4 points.
- A response earns the fifth point only for an answer of $H(t) = 1 + 3e^{\sin(t/2)}$ or a mathematically equivalent expression for $H(t)$ such as $H(t) = 1 + e^{\sin(t/2)+\ln 3}$.
- A response does not need to argue that $|H-1| = H-1$ in order to earn the fifth point.
- Special case: A response that presents an incorrect separation of variables of $\frac{1}{2} \cdot \frac{dH}{H-1} = \cos\left(\frac{t}{2}\right) dt$ does not earn the first point or the fifth point but is eligible for the 2 antiderivative points. If the response earns at least 1 of the 2 antiderivative points, then the response is eligible for the fourth point.

Total for part (c) 5 points

Sample: How many points would you award?

2024 Sample 1c

Answer

$\frac{dH}{H-1} = \frac{1}{2}\cos\left(\frac{t}{2}\right)dt$	Separation of variables	**1 point**
$\int \frac{dH}{H-1} = \int \frac{1}{2}\cos\left(\frac{t}{2}\right)dt$	One antiderivative	**1 point**
$\Rightarrow \ln\lvert H-1\rvert = \sin\left(\frac{t}{2}\right) + C$	Second antiderivative	**1 point**
$\ln\lvert 4-1\rvert = \sin\left(\frac{0}{2}\right) + C \Rightarrow C = \ln 3$ Because $H(0) = 4$, $H > 1$, so $\lvert H-1\rvert = H-1$. $\ln(H-1) = \sin\left(\frac{t}{2}\right) + \ln 3$	Constant of integration and uses initial condition	**1 point**
$H - 1 = e^{\sin(t/2)+\ln 3} = 3e^{\sin(t/2)}$ $H(t) = 1 + 3e^{\sin(t/2)}$	Solves for H	**1 point**

Response for question 3(c)

$$\frac{dH}{dt} = \frac{1}{2}(H-1)\cos\left(\frac{t}{2}\right)$$

$$dH \cdot \frac{1}{\frac{1}{2}(H-1)} = \cos\left(\frac{t}{2}\right)dt$$

$$dH \cdot \frac{2}{H-1} = \cos\left(\frac{t}{2}\right)dt$$

$$\int \frac{2\,dH}{H-1} = \int \cos\left(\frac{t}{2}\right)dt$$

$$2\ln\lvert H-1\rvert = 2\sin\left(\frac{t}{2}\right) + C$$

$$2\ln\lvert 3\rvert = 2\sin(0) + C$$

$$2\ln\lvert 3\rvert = C$$

$$\ln\lvert H-1\rvert = \sin\left(\frac{t}{2}\right) + \ln\lvert 3\rvert$$

$$H - 1 = e^{\sin\frac{t}{2} + \ln 3}$$

$$H = e^{\sin\frac{t}{2} + \ln 3} + 1$$

Actual Score: 5

2024 Sample 2c

Sample: How many points would you award?

Answer

$\frac{dH}{H-1} = \frac{1}{2}\cos\left(\frac{t}{2}\right)dt$	Separation of variables	**1 point**
$\int \frac{dH}{H-1} = \int \frac{1}{2}\cos\left(\frac{t}{2}\right) dt$	One antiderivative	**1 point**
$\Rightarrow \ln\lvert H-1\rvert = \sin\left(\frac{t}{2}\right) + C$	Second antiderivative	**1 point**
$\ln\lvert 4-1\rvert = \sin\left(\frac{0}{2}\right) + C \Rightarrow C = \ln 3$ Because $H(0) = 4,\ H > 1$, so $\lvert H-1\rvert = H-1$. $\ln(H-1) = \sin\left(\frac{t}{2}\right) + \ln 3$	Constant of integration and uses initial condition	**1 point**
$H-1 = e^{\sin(t/2)+\ln 3} = 3e^{\sin(t/2)}$ $H(t) = 1 + 3e^{\sin(t/2)}$	Solves for H	**1 point**

Response for question 3(c)

$$\frac{dH}{dt} = \frac{1}{2}(H-1)\cos\left(\frac{t}{2}\right)$$

$$\int \frac{1}{H-1}\,dH = \int \frac{1}{2}\cos\left(\frac{t}{2}\right)dt \qquad u = \frac{t}{2}$$

$$\int \frac{1}{H-1}\,dH = \frac{1}{2}\int \cos(u)\,\frac{du}{2} \qquad \frac{du}{2} = dt$$

$$\ln\lvert H-1\rvert = \frac{1}{4}\sin\left(\frac{t}{2}\right) + C$$

$$\ln\lvert 4-1\rvert = \frac{1}{4}\sin(0) + C$$

$$\ln\lvert 3\rvert = C$$

$$e^{(\ln\lvert H-1\rvert)} = e^{\left(\frac{1}{4}\sin\left(\frac{t}{2}\right) + \ln\lvert 3\rvert\right)}$$

$$\lvert H-1\rvert = 3e^{\frac{1}{4}\sin\left(\frac{t}{2}\right)}$$

$$\boxed{H = 3e^{\frac{1}{4}\sin\left(\frac{t}{2}\right)} + 1}$$

Actual Score: 3

2024 Sample 3c

Sample: How many points would you award?

Answer

$\dfrac{dH}{H-1} = \dfrac{1}{2}\cos\left(\dfrac{t}{2}\right)dt$	Separation of variables	**1 point**
$\displaystyle\int \dfrac{dH}{H-1} = \int \dfrac{1}{2}\cos\left(\dfrac{t}{2}\right) dt$ $\Rightarrow \ln\lvert H-1\rvert = \sin\left(\dfrac{t}{2}\right) + C$	One antiderivative	**1 point**
	Second antiderivative	**1 point**
$\ln\lvert 4-1\rvert = \sin\left(\dfrac{0}{2}\right) + C \Rightarrow C = \ln 3$ Because $H(0) = 4$, $H > 1$, so $\lvert H-1\rvert = H-1$. $\ln(H-1) = \sin\left(\dfrac{t}{2}\right) + \ln 3$	Constant of integration and uses initial condition	**1 point**
$H-1 = e^{\sin(t/2)+\ln 3} = 3e^{\sin(t/2)}$ $H(t) = 1 + 3e^{\sin(t/2)}$	Solves for H	**1 point**

$H(0)=4$

$$\frac{dH}{dt} = \frac{1}{2}(H-1)\cos\left(\frac{t}{2}\right)$$

$$\int \frac{dH}{(H-1)} = \int \frac{1}{2}\cos\left(\frac{t}{2}\right)$$

$$\int \frac{1}{2}\sin\left(\frac{0}{2}\right)$$

$$\sin(0)$$

$$1$$

$$\frac{1}{2}\cdot 1 + C$$

$$\ln|H-1| = \frac{1}{2} + C$$

$$\ln|0-1|$$

$$\ln|-1|$$

$$1 = \frac{1}{2} + C$$

$$1 - \frac{1}{2} + C = 4$$

$$\frac{2}{2} - \frac{1}{2}$$

$$= \frac{1}{2} + C = 4 - \frac{1}{2}$$

$$C = \frac{8}{2} - \frac{1}{2}$$

$$\boxed{C = \frac{7}{2}}$$

Actual Score: 2

Random Question